Dislocations and Bragg glasses in two dimensions

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We discuss the question of the generation of topological defects (dislocations) by quenched disorder in two dimensional periodic systems. In a previous study [Phys. Rev. B **52** 1242 (1995)] we found that, contrarily to $d = 3$, unpaired dislocations appear in $d = 2$ above a length scale ξ_D, which we estimated. We extend this description to include effects of freezing and pinning of dislocations at low temperature. The resulting ξ_D at low temperature is found to be *larger* than our previous estimate, which is recovered above a characteristic temperature. The dependence of ξ_D in the bare core energy of dislocations is a stretched exponential. We stress that for all temperatures below melting ξ_D becomes arbitrarily large at weak disorder compared to the translational order length $R_a \gg a$. Thus there is a wide region of length scales, temperature and disorder where Bragg glass like behavior should be observable.

An outstanding current problem in condensed matter physics is to understand the physical properties of periodic systems in presence of point impurities. This is important for numerous experimental systems, such as vortices in type II superconductors[1,2], Wigner crystallization of electrons[3–6], charge density waves[7] etc.. We have proposed[8] that, contrarily to previous claims, such systems generically possess at weak disorder in three dimensions a distinct thermodynamic glass phase, the Bragg glass, with perfect topological order (dislocation free) and quasi long range translational order. An immediate consequence of this theory is that a phase transition away from the Bragg glass at which dislocations suddenly proliferate must occur upon increase of disorder (or field) which is determinant for the phase diagram of type II superconductors[8,9]. Thus, whether dislocations are generated or not by disorder has emerged as a central question for these systems. In three dimensions there is increasing theoretical[10–12] , numerical[13–15] and experimental evidence[16–19] that this topologically ordered Bragg glass phase exists. By contrast, the strong disorder glassy state in three dimensional superconductors is still poorly known beyond the fact that it must contain topological defects. It is yet unclear whether this state is a distinct thermodynamic phase or simply a crossover from a pinned liquid. The initial proposal of a "vortex glass" phase[20,21] (claimed to contain topological defects) now clearly cannot stand at weak disorder. It is even unclear, in view of recent numerical[22,23] and experimental data[24] whether such a phase, as described in Ref. 20,21, exists at all in real (i.e with screening) three dimensional superconductors.

In two dimensions it is easier to generate dislocations and we have obtained[8] that within the conventional perturbative RG analysis unpaired dislocations appear beyond a length scale ξ_D which we have estimated. A similar result was also obtained for the specific case of two dimensional triangular lattice[25,26]. Recent numerical studies[13,27] in $d = 2$ support these predictions. Note however that the question of the stability of the Bragg glass can only be decided numerically by investigating the *weak disorder* region. The RG flow used[8], although based on perturbation theory[28,29], already captured the delicate balance between the elastic energy cost and the energy gain due to disorder. One striking consequence was that although unpaired dislocations are generated, the scale at which they appear, obtained as:

$$\xi_D \sim R_a e^{c(\ln(R_a/a))^{1/2}} \tag{1}$$

can be made, at weak disorder $R_a \gg a$, arbitrarily large compared to the length R_a at which the displacements become of the order of the lattice spacing.

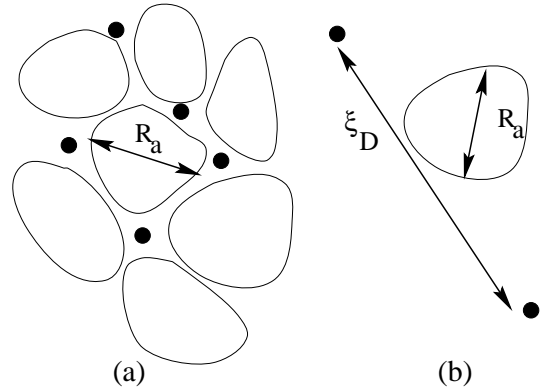(a)   (b)

FIG. 1.   (i) Conventional picture of a solid broken up by disorder in domains of size R_a with unpaired dislocations (black dots) appearing at the same scale. (ii) correct picture for weak disorder: the scale ξ_D at which unpaired dislocations appear is larger than the scale R_a at which translational order starts decaying slowly.

Below melting we thus obtained that even in $d = 2$ the naive picture of the system as breaking up in crystallites of size R_a is still incorrect as illustrated in Fig. 1. Even if $d = 2$ is the lower critical dimension of the Bragg glass, since ξ_D can be very large in practice a wide regime of effective Bragg glass behaviour should still be observable. As pointed out in Ref. 25 where a theory predicting $\xi_D(T)$ near melting was obtained, this may allow to understand why experiments on two dimensional superconductors[30] show a sharp change of behaviour with characteristics of melting.

In this paper, we further examine the two dimensional problem. Indeed there has been recent evidence that at low temperature the conventional RG equations can *overestimate* the importance of dislocations in related models[31−36,25,37,38]. Thus our initial estimate (1) of ξ_D, which should be accurate at intermediate temperatures, may become only a lower bound on ξ_D at low temperature. We give here a more accurate description of the low temperature properties. This is of importance for applications to experiments or simulations on classical systems, often performed at low temperature but also for ground state properties of quantum disordered systems, such as the two dimensional Wigner crystal[39,40].

In order to study dislocation problems in $d = 2$ let us restrict for simplicity to the single component scalar XY model in a random field, defined in the continuum by the partition sum $Z = \int D\phi e^{-H/T}$ and hamiltonian:

$$H = \int d^2x \, \frac{J}{2\pi} (\nabla\phi(x) - \eta(x))^2 \tag{2}$$
$$- \zeta_1(x)\cos\phi(x) - \zeta_2(x)\sin\phi(x)$$

with $\overline{\eta_i(x)\eta_j(x')} = \pi\sigma\delta_{ij}\delta(x - x')$ and $\overline{\zeta_i(x)\zeta_j(x')} = g\delta_{ij}\delta(x - x')$ are Gaussian white noises. Configurations with vortices (of integer charge q_i at x_i) are described by decomposing $\phi(x) = \phi_{SW}(x) + \phi_V(x)$ where $\phi_{SW}(x)$ is the smooth (spin wave) field and $\phi_V(x)$ the vortex contribution $\nabla \times \nabla\phi_V(x) = \sum_i q_i 2\pi\delta(x - x_i)$. This model contains essential ingredients of a variety of elastic disordered systems[8] in $d = 2$. For instance, in the case of a lattice, $\phi = 2\pi u/a$ can be thought of as a displacement field u in units of lattice spacing a. Point like impurities produce a random potential $V(x)$ which couples to the density and leads to (2) with $g \sim \Delta_{K_0}/a^4$ proportional to the amplitude of the disorder with Fourier component close to $K_0 = 2\pi/a$ (called the pinning disorder) and $\sigma \sim \Delta_0/4\pi J^2 a^2$ proportional to the long wavelength disorder, where $\Delta_q = \overline{V_q V_{-q}}$. Vortices in the field ϕ correspond to dislocations in the lattice and thus will generically be called "dislocations" in the following. These defects are characterized by a fugacity y, which in the bare model is related to the defect core energy $y = e^{-E_c/T}$. Note that the long wavelength disorder σ is always generated by coarse graining. The bare model (2) corresponds, for the lattice problem, to the simpler case where the correlation length of the disorder r_f is of the order of the lattice spacing a. Thus in the model

specifically studied in this paper, the translational order correlation length[8] R_a - such that relative displacements $u(R_a) - u(0) \sim a$, is of the same order than the Larkin Ovchinnikov[41] pinning length R_c - such that relative displacements $u(R_c) - u(0) \sim r_f$, and we will implicitly equate them in the following. The situation $r_f \ll a$ (thus $R_c \ll R_a$) will be briefly mentioned at the end.

Let us summarize the RG analysis which led to the estimate in Ref. 8 of the scale ξ_D beyond which unpaired dislocations appear in $d = 2$ at weak disorder. The RG equations for the fugacity of dislocations, the disorder and the stiffness were derived by Cardy and Ostlund (CO)[28,29]. The fugacity of the vortices y satisfies to lowest order in y:

$$\frac{dy}{dl} = (2 - \frac{J}{T} + \frac{\sigma J^2}{T^2}) \, y \tag{3}$$

with $T_m = J/2$ is the pure system Kosterlitz-Thouless transition temperature and $l = \ln L$ is the logarithmic scale. The pinning disorder renormalizes as:

$$\frac{dg}{dl} = 2\tau g - Bg^2 \tag{4}$$

up to $O(g^3)$ terms, with $\tau = 1 - \frac{T}{T_g}$, $T_g = 4J = 8T_m$ and B a nonuniversal constant. If dislocations are excluded by hand (setting $y = 0$) there is a transition at $T = T_g$ between a high temperature phase ($T > T_g$) where the disorder is irrelevant and a low temperature glass phase ($T < T_g$). For $T < T_g$ there is a line of fixed points $g = g^*$ which describes a 2d Bragg glass phase where displacements grow as $u \sim \ln x$ and beyond which translational order slowly decays. This asymptotic behaviour is reached at the length $R_a \sim a(\frac{g^*}{g_0})^{\frac{1}{2\tau}}$ (where g_0 is the bare value) which for weak disorder is $R_a \gg a$. The global structure of the RG (together with functional RG extensions) suggests that the fixed line continues down to $T = 0$ and that correlations behave as $\overline{\langle(\phi(x) - \phi(0))^2\rangle} = C(T)\ln^2(x/R_a)$ with a T dependent prefactor[42]. It is perturbatively controlled only near T_g, where g^* is small, and the correct universal prefactor[43] $C(T) = 2\tau^2$ was derived[26] to lowest order in τ. Numerical simulations have measured[44] $C(T)$ near T_g[45] and at $T = 0$[46,47,27]

When dislocations are allowed in the model, one finds that they are perturbatively relevant above T_m leading to a liquid. Below T_m however, since the bare value of σ is small for weak disorder, one could naively conclude from (3) that dislocations are suppressed at low enough temperature. As was discussed in Ref. 8 this is not the case and in fact the 2d Bragg glass fixed point is *unstable* to dislocations. Such an instability is a peculiar feature of $d = 2$ and does not occur in $d = 3$ where the Bragg glass phase is *stable* with respect to dislocations at weak disorder. The instability occurs because in $d = 2$ the long wavelength disorder σ is also renormalized:

$$\frac{d\sigma}{dl} = Ag^2 \tag{5}$$

to lowest order (with $A = B^2$ at $T = T_g$) and thus grows unboundedly with the scale as $\sigma(l) \sim \sigma^* + 2C(T) \ln(L/R_a)$ for $L > R_a$, with $2C(T) = Ag^{*2} = 4\tau^2$ near T_g. As can be seen from (3) $y(l)$ starts increasing beyond a certain length scale and eventually becomes of order $y(l_D) \sim 1$ at a scale $l_D = \ln(\xi_D/a)$. At that scale unpaired dislocations dominate the behaviour and translational order is exponentially destroyed beyond ξ_D.
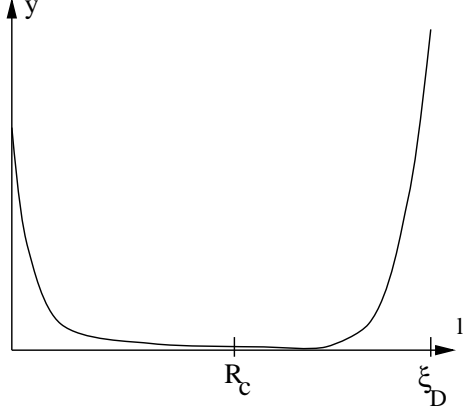

FIG. 2. Dependence of the dislocation fugacity y in the length scale for weak disorder. It is first strongly renormalized downwards before it eventually shoots up again and reaches values of order unity at $\xi_D \gg R_a$.

However, ξ_D can be very large and in particular much larger than R_a. This is because for weak disorder and $T < T_m$ the fugacity of dislocations is first strongly renormalized downwards for scales smaller than R_a (see (3)), as shown in Figure 2. Integrating the flow led us to the estimate[8]:

$$\xi_D \sim R_a e^{c\sqrt{2(\frac{T_m(\sigma)}{T} - 1)\ln(R_a/a)}} \qquad (6)$$

where $T_m(\sigma)$ is the boundary of the XY phase in the absence of pinning disorder ($g = 0$) as given by the CO equations[29] (see Fig. 4 below). The case of triangular lattices in presence of disorder was studied in more details recently using a $N = 2$ component model, and a generalization of both the above CO equations (3,4,5) and the KTNHY equations (which describe the fusion of pure crystals) was obtained[25,48]. It confirms that a similar estimate as (6) holds for lattices and yields a complementary formula for ξ_D around the pure crystal melting transition.

To obtain (6) the only assumption made in Ref. 8 and supported by the structure of the RG flow, was that beyond the scale R_a pinning disorder g has reached a fixed point and that $\sigma(l)$ grows as $l = \ln L$. This is equivalent to use beyond R_a the effective "random stress" model which reads in the continuum:

$$H = \int d^2x \frac{J}{2\pi} (\nabla\phi(x) - \eta(x))^2 \qquad (7)$$

to which one must add vortices as discussed above. The disorder $\overline{\eta_i(q)\eta_j(-q)} = \pi\sigma(q)\delta_{ij}$, with $\sigma(q) \sim \ln(1/q)$,

now depends on the scale in a logarithmic way as $\sigma(l) \sim \sigma^* + 2C(T) \ln(L/R_a)$, where $C(T)$ is the amplitude defined above. A recent numerical work[27] comes as additional support that this assumption is indeed valid. When $\sigma(l) = \sigma$ is scale independent, this model reduces to the "random phase shift model"[29]. The advantage of model (7) is that it can be treated by RG or by qualitative arguments even in presence of dislocations.

Let us now reexamine (7) at low temperature. Because of the reentrance of the disordered phase present in the CO equations[29] an extrapolation of (6) would lead to a small ξ_D at low temperature (below $T_m^-(\sigma)$ in Fig. 4). However, the original CO equations tend to *overestimate* the effect of dislocations as they neglect non thermalization and pinning of dislocations[8]. Indeed, recent reexamination of the phase diagram of the random phase shift model (7) has shown that the reentrant disordered phase[29,28] disappears when these effects are taken into account[31]. Although in the high temperature region $T > T_m/2$ the RG equation (3) was found to be correct, new physics arises below the line $T < T^*(\sigma)$. In order to reexamine our previous estimate (6) for ξ_D at low temperature, we extend to the present case (where $\sigma(l)$ depends logarithmically on L) the modified RG analysis recently developed for the random phase shift model[31,33−36,48].

To study the relevance of dislocations it is first useful to consider a single dislocation or dipole at $T = 0$. The simplest energy argument, presented first in Ref. 31,49, estimates when it is favorable to place one vortex or a dipole in the system of size L. This vortex sees a 2d random potential $V(x)$ with logarithmic correlations, and one must thus estimate the minimum energy E_{min} of this random potential. A reasonable approximation is obtained by neglecting correlations but keeping the correct local variance as[49]:

$$\frac{1}{L^2} \sim \int_{-\infty}^{E_{min}} \frac{dV}{\sqrt{4\pi\sigma J^2 \ln L}} \exp(-\frac{V^2}{4\sigma J^2 \ln L}) \qquad (8)$$

This estimate leads to $E_{min} \sim -\sqrt{8\sigma}J\ln L$. The more accurate methods which are now available to take correlations into account[50,36] confirm that the prefactor is exact, and estimate the (large) corrections to scaling. The energy gain by disorder thus overcomes the elastic cost $E_{el} = J\ln L$ for $\sigma > \sigma_c = 1/8$. Below this value an XY phase, dislocation free at large scale, exists contrarily to the earlier conclusion[29] based on (3). Thus these low temperature effects strongly reduce the relevance of dislocations. In the random phase shift model, the topologically ordered Bragg glass phase is thermodynamically stable at low temperatures.

One can thus expect a similar *reduction* of the importance of dislocations in the presence of pinning disorder, i.e for the random stress model with $\sigma(l) \sim \ln L$ compared to our previous estimates based on the CO equations. A straightforward modification of the above argument (8), replacing σ by $\sigma(l)$ in (8) leads to $E_{min} \sim$

$\sqrt{8\sigma(l)}J\ln L \sim (\ln L)^{3/2}$. The energy gain due to disorder now always overcomes the elastic cost $J\ln L$ at large scale and dislocations are always generated in model (7). This modification of the argument of Ref. 49 thus allows to recover in a very simple way the $(\ln L)^{3/2}$ estimate[27] for the optimal energy of a dislocation pair.

However these types of energy arguments or the analysis[27] for a *single* dislocation (or dipole) does not by itself allow to compute the length scale ξ_D at which vortices (dislocations) destroy XY (positional) order. Indeed, to destroy the order exponentially one needs a *finite density* of defects. In particular it would be *incorrect* to identify ξ_D as the length scale at which the disorder energy becomes of the order of the elastic energy. This is clear for instance when looking at the KT transition were elastic energy is $J\ln L$ and entropy is $2T\ln L$. When $T > T_m = J/2$ balancing only these two terms would incorrectly predict unpaired dislocations of size a near T_m, when in reality they occur at a much larger length scale $\xi_m \sim ae^{-c(T-T_m)^{-1/2}}$. To get the correct result one must take into account both the defect core energy E_c and the screening by smaller dipoles. The same effects *must* be also taken into account for the random stress model in order to quantify the importance of dislocations.

We now estimate the length scale ξ_D at low temperature using a RG analysis for the random stress model (7). In a recent work[36] it was shown that to describe the correct physics in the model with constant $\sigma(l) = \sigma$, one must follow the full distribution of dislocation core energies $P_l(E_c)$, found to satisfy a non linear RG equation. At $T = 0$, this non linear RG equation solves the problem of energy minimization iteratively on successive logarithmic scales. Its solution $P_l(E_c)$, develops broad tails at low temperature, as illustrated in Fig. 3. Very schematically, while the center of the distribution is biased towards the right as $\sim J\ln L$, the width grows as $\sim \sigma J^2 \ln L$. For large enough σ ($\sigma > 1/8$) the width "wins" and the probability of finding a site with negative effective core energy eventually increases with the scale. This analysis can be extended in an accurate manner to the present case, by studying the non linear RG equation in presence of a scale dependent $\sigma(l) \sim l$.

For our present purpose it is enough to use the following simplified version where $P_l(E_c)$ at scale l is given by:

$$P_l(E_c) \sim \exp(2l - \frac{(E_c - Jl - E_c^0)^2}{4J^2 \int_0^l \sigma(l')dl' + c}) \qquad (9)$$

This amounts to study the solution of the linearized version of the RG equation[36] which should be a good approximation in the small E_c far tail of the distribution of $P_l(E_c)$ needed here. The distance between unpaired dislocations corresponds to the length scale $l = l_D = \ln(\xi_D/a)$ at which $P_l(0) \sim 1$. Physically this corresponds, in the renormalized model, to putting one dislocation per site. The above form, for constant σ, clearly yields a dislocation free phase ($l_D = +\infty$) for $\sigma < 1/8$ and prolifer-

ation of dislocations for $\sigma > 1/8$. In that case differentiating $P_l(0)$ from (9) yields back the RG equation[31] for the "effective dipole fugacity" y^2 as:

$$\frac{dy}{dl} = (2 - \frac{1}{4\sigma}) y \qquad T < T^* = 2\sigma J \qquad (10)$$

identified as $y^2 \sim P_l(0)^2$ below the line $T = T* = 2\sigma J$ where the freezing does occur (see Figure 3).
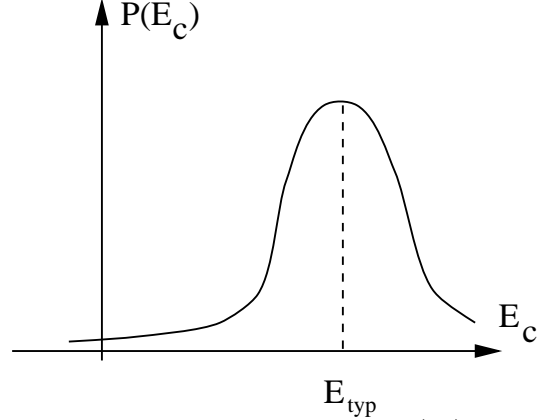

FIG. 3. scale dependent distribution $P_l(E_c)$ of the effective core energy as discussed in Ref. 36. The bulk of the distribution is centered around a typical value $E_{typ}(l)$ but broad tails develop at low temperature

In the present case the above equation (9) is complemented by the equation for $\sigma(l)$ which arises as the solution of (4,5). Denoting $R_a = ae^{l_a}$, and using the simplified form $g(l) = g^*e^{2\tau(l-l_a)}$ for $l < l_a$ and $g(l) = g^*$ for $l > l_a$, we obtain:

$$\sigma(l) = \sigma_0 + \frac{Ag^{*2}}{4\tau}(e^{4\tau(l-l_a)} - e^{-4\tau l_a}) \qquad l < l_a$$

$$\sigma(l) \approx \sigma^* + Ag^{*2}(l - l_a) \qquad l > l_a \qquad (11)$$

with $\sigma^* = \frac{Ag^{*2}}{4\tau}$. This is valid near T_g, but can be extended everywhere by replacing Ag^{*2} by $2C(T)$ in (11). Also, in the range of length scales needed here, and far from the critical crossover region $\sigma_0 = 1/8$, we can neglect the renormalization of σ and J by dislocations. Using (11) , (9) leads to:

$$\xi_D \sim R_a e^{c\sqrt{(\frac{1}{8} - \sigma_0)\ln(R_a/a)}} \qquad (12)$$

with $c \approx 1/\sqrt{C(T)}$. This expression holds in the low temperature region $T < T^* = 2\sigma J$ and for weak disorder $R_a \gg a$. Interestingly, it does have a form very similar to (6) which is valid in the high temperature region $T^* < T < T_m(\sigma)$ in Fig. 4, but for the fact that now the disorder σ_0 plays a role analogous to temperature. The expression (12) smoothly connects with our previous estimate (6) at higher temperature. It would be interesting to check the predictions (6,12) in numerical simulations.

A previous numerical work[13] on the random XY model close to T_m is indeed consistent with ξ_D larger than R_a although no attempts were made to compare the results with (6). Finally, very near $\sigma_0 = 1/8$ we expect that ξ_D will take a critical crossover scaling form as is the case[25] near $T = T_m$.
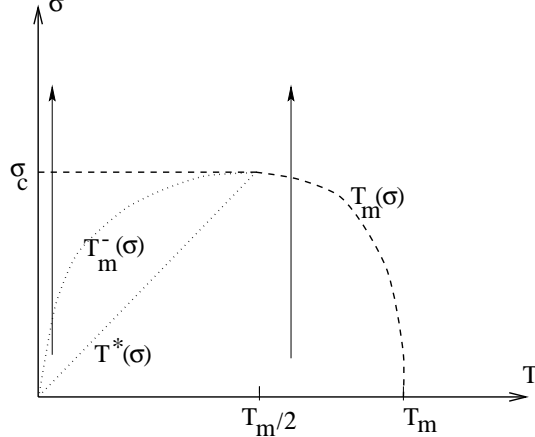

FIG. 4. Different regions of the temperature T, disorder σ phase diagram. The dashed lines represent true XY phase boundaries ($T_m(\sigma)$ and $\sigma = \sigma_c = 1/8$) for the model without pinning disorder $g = 0$. The dotted lines are the old (incorrect) XY boundary $T_m^-(\sigma)$ obtained via CO-RG, and the new freezing line $T^*(\sigma)$. In presence of pinning disorder $g > 0$, $\sigma(l)$ grows with the scale and eventually dislocations appear. The arrowed lines represent the RG trajectories in the two cases (low and intermediate temperature) discussed in the text

Note that it would be tempting to replace directly σ by $\sigma(l)$ in (10) and integrate the flow. This is incorrect, as can be seen by writing the RG equation for $P_l(0)$ using (9). This is a manifestation of the fact that one must here follow a distribution which cannot, at low temperature, be parametrized by a single fugacity-like parameter y.

In numerical simulations it may be easier to study intermediate disorder but change artificially the bare core energy E_c^0. Keeping the core energy dependence in the previous analysis of (9) leads to the following equation for $l_D = \ln(\xi_D/a)$:

$$0 = 2l_D - \frac{(Jl_D + E_c^0)^2}{4J^2 C(T) l_D^2} \tag{13}$$

In the limit of large E_c^0 this gives:

$$\xi_D \sim a \; e^{\left(\frac{E_c^0}{E^*}\right)^{\frac{2}{3}}} \tag{14}$$

with $E^* \approx 2\sqrt{2C(T)}J$. It is noteworthy that a very recent numerical work[51] finds a similar law with an exponent ~ 0.7. The above result can be written $E_c^0 \sim (\ln \xi_D)^{3/2}$ in agreement with the general scaling of optimal disorder energies in this problem.

Let us close with an overall perspective on the situation in $d = 2$. The above analysis of the flow of pinning disorder can be improved, e.g. allowing for more general RG flow structure or using functional RG techniques appropriate to zero temperature (where the model (2) develops higher harmonics and nonanalyticity). This, we expect should not change the results obtained here, up to numerical prefactors, as long as the approximation introduced in Ref. 8 by the random stress model holds at large scale. This model allows to treat some of the conventionally non perturbative effects related to dislocation freezing by long wavelength disorder. Proving its validity everywhere, or going beyond it, is difficult analytically, and challenging numerically because of large corrections to scaling. Thus it still cannot be excluded that more surprises may be in store when further non perturbative effects (e.g. the effect of pinning disorder g on dislocations) are taken into account. The case $r_f \ll a$ can also be studied. An intermediate "random manifold regime" then exists between $R_c < L < R_a \sim R_c(a/r_f)^\alpha$ with $\alpha \sim 3$. A (functional) RG analysis can be sketched. The RG equation for σ becomes $\partial_l \sigma \sim \sum_K K^2 g_K$ to lowest order, while the g_K reach a crossover functional form beyond $L = R_c$ and another, asymptotic one, beyond $L = R_a$. Although $\sigma(l)$ starts growing beyond R_c, we find that it remains small until R_a. This indicates that the above results should still hold.

To conclude we have analyzed the question of the generation of dislocations by disorder in $d = 2$. In an ealier study[8] based on a random stress model approximation of the Cardy Ostlund equations we had found that contrarily to $d = 3$, in $d = 2$ unpaired dislocations appear beyond a length scale ξ_D. Recent numerical simulations[13,27] have reached a similar conclusion. At low temperature we have improved our estimate for ξ_D taking into account freezing of dislocations by long wavelength disorder. Taking into account these effects increases ξ_D compared to our previous estimates at low temperature and smoothly connects to it at higher temperature. Thus the range of length scales where the system behaves *effectively* as a Bragg glass is wider than previously expected. We also computed the bare core energy dependence of ξ_D which exhibits stretched exponential behaviour. It would be interesting to further explore numerically these systems, particularly at weak disorder.

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